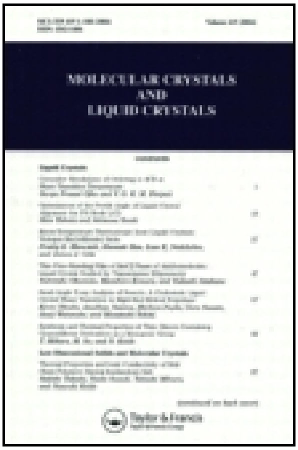

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Optical Properties of a Chiral Liquid Crystal in a Generalized Coupled Mode Formalism

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Optical Properties of a Chiral Liquid Crystal in a Generalized Coupled Mode Formalism

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We propose a general formalism suitable for calculating the propagation of the optical normal modes through a chiral liquid crystal as one of the characteristic parameters varies. Based on the fact that the normal modes are the eigenstates of the generator of translation (the wave number operator **K**), the change in **K** is evaluated by means of a characteristic parameter such as chirality. The numerical calculations are in good agreement with the analytic results.

Keywords: chiral liquid crystal; optical activity; coupled mode

INTRODUCTION

In classical electrodynamics, the normal modes can be described in terms of non-trivial solutions of the Maxwell equations coupled with the dielectric tensor representing a given material. However, for a material with a complex structure, complicated calculations are involved to understand the essential physics associated with the normal modes. In this work, we present an alternative formalism for obtaining the normal modes of a cholesteric liquid crystal (ChLC) by using the fact that the infinitesimally evolved chirality influences the normal modes. In this approach, we construct the generator of chirality (**Q**) and use the Baker-Hausdorff lemma to evaluate the change of **K** from the viewpoint of the Heisenberg picture.

THEORY

A normal mode is defined as a state which preserves the initial state in the course of translation but having only phase change. This definition requires the normal modes to be the eigenstates of **K** which is the generator of translation [1]. Therefore, a diagonal matrix representation of **K** can be described in terms of the normal modes. For example, a nematic liquid crystal (NLC) has two normal modes ($|\xi\rangle$ and $|\eta\rangle$), having the wave-numbers of βe and βo, respectively. The operator **K** describing the light propagation through NLC ($\mathbf{K}_n$) can be expressed as

$$\mathbf{K}_n = \beta e|\xi\rangle\langle\xi| - \beta o|\eta\rangle\langle\eta| . \tag{1}$$

For convenience, the minus sign is adopted in βo.

Now we consider the case where a helical structure associated with the director of a LC (**n**) forms along a certain axis such that the value of 2π/pitch has a nonzero value of q. We will construct a finite chirality operator which represents the effects of the appearance of a helical structure on the liqht propagation. A finite chirality operator is then given as exp[-i**Q**q] where **Q** is the generator of chirality. From the viewpoint of the Heisenberg picture, we examine the normal modes by observing the change in the eigenstates of **K**.

$$\mathbf{K} = \exp[-i\mathbf{Q}q]^{\dagger}\,\mathbf{K}_n \exp[-i\mathbf{Q}q]. \tag{2}$$

We now derive a definite form of the operator **Q**. There is no formal way of deriving the generator. Thus, we construct the structure of **Q** by considering the origin of chirality effects. Suppose that the infinitesimal chirality tends to rotate two principal axes of LC (ξ-axis and η-axis) by an angle of $d\phi$. The resultant chirality dq, equivalent to $d\phi$, does not affect the propagation of the individual states, $|\xi\rangle$ and $|\eta\rangle$. However, it redistributes the two states in the following manner;

$$1 - i\mathbf{Q}dq \sim [-\sin(d\phi)|\eta\rangle + \cos(d\phi)|\xi\rangle]\langle\xi| + [\sin(d\phi)|\xi\rangle + \cos(d\phi)|\eta\rangle]\langle\eta| \qquad (3)$$

By the rotation of $d\phi$, the projection of $|\xi\rangle$ exits in the η-axis. Note that no such component appears in the case of NLC. A part of $|\xi\rangle$ experiences the transition to $|\eta\rangle$ and vice versa. This transition is physically equivalent to the case that occurs between two isotropic materials. Therefore, no phase evolution is involved. It is convenient to introduce another form of a finite chirality operator, $\exp(-i\Phi\phi)$. Note that the generator of rotation (Φ) has the dimension of the angular momentum. Therefore, as the result of a transition, the angular momentum is transferred. The transferred angular momentum (**a**) is the difference of the angular momentum between $|\xi\rangle$ and $|\eta\rangle$, and it is written as $\mathbf{a} = [\Delta z'(\beta e+\beta o)]^{-1}$. With the help of these arguments, the complete form of **Q** can be written as

$$\mathbf{Q} = (\beta e+\beta o)^{-1}[\,|\eta\rangle\langle\xi| + |\xi\rangle\langle\eta|\,] \qquad (4)$$

NUMERICAL CALCULATIONS

Now, let us evaluate the operator K with chirality using Eq. (2). For this purpose, the well-known Baker-Hausdorff lemma is used.

$$\begin{aligned}\mathbf{K} &= \exp[-i\mathbf{Q}q]^{\dagger}\,\mathbf{K}_n \exp[-i\mathbf{Q}q] \\ &= \mathbf{K}_n + iq[Q, \mathbf{K}_n] + (iq)^2/2![Q, [Q, \mathbf{K}_n]] + \ldots \end{aligned} \qquad (5)$$

For the normal incidence of light onto ChLC, keeping the terms up to the first order in q, the obtained numerical results agree with the exact analytic results [2]. In the numerical calculation, we use the value of

1.7 and 1.5 as the refractive index of NLC, pertinent to $|\xi>$ and $|\eta>$ respectively.

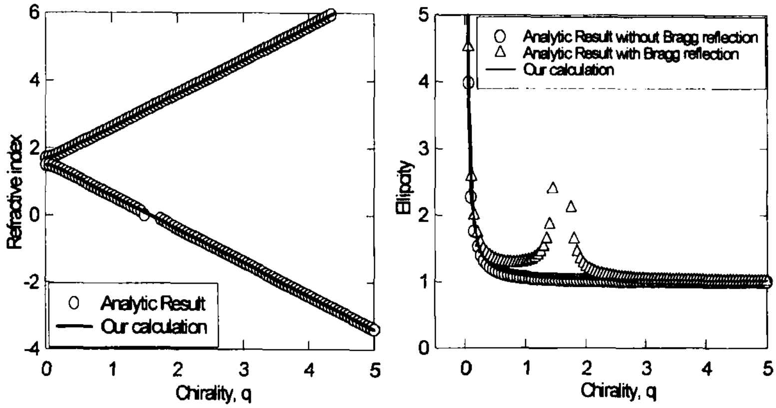

Fig. 1 Comparison between numerical calculations and analytic results

Fig. 1 shows the refractive index and the ellipcity (ratio of long axis to short axis) as a function of the scaled q by (ω/c). As the pitch (or q) approaches the wavelength, a selective Bragg reflection takes place. In our case, this behavior would be expected when the contra-directional transfer of **a** is included. Outside the Bragg regime, our calculations show an excellent agreement.

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